REEL-C 7377

A.T.I 208429

UNCLASSIFIED

AD-137355

OFFICE OF NAVAL RESEARCH

CONTRACT N6 ONR-262, TASK ORDER I

NR 014 401

TECHNICAL REPORT NO. 8

A GENERALIZED THEORY OF THE CRYSTAL TRANSMITTER AND RECEIVER FOR PLANE WAVES

NOVEMBER 20, 1950

SUBMITTED BY

WALTER G. CADY, PROJECT DIRECTOR

3

SCOTT LABORATORY OF PHYSICS
WESLEYAN UNIVERSITY MIDDLETOWN, CONN.

COPY NO 16

OFFICE OF NAVAL RESEARCH

CONTRACT N6onr-262, TASK ORDER I

NR 014 401

Technical Report No. 8

A GENERALIZED THEORY OF THE CRYSTAL TRANSMITTER AND RECEIVER FOR PLANE WAVES

By

Walter G. Cady

November 20, 1950

Submitted by

Walter G. Cady, Project Director

Scott Laboratory of Physics

Wesleyan University Middletown, Connecticut

ABSTRACT

From a consideration of boundary conditions a set of equations is derived for a plane-wave crystal transducer operated as either a transmitter or a receiver. The transducer may be of either the lengthwise or thickness type, and the crystals, back plate, and front plate, may have any dimensions in the wave-direction.

The equations are applied especially to the problem of the receiver. The tuned receiver, in which the transducer is in resonance with the incident radiation, receives particular attention. Expressions are given for voltage and power in the output circuit as functions of the output admittance, taking account also of losses in the transducer. From these expressions the output conductance and susceptance for maximal power are calculated. It is shown that, in an ideal no-loss transducer, all the incident power could be converted into useful output, the transducer becoming a perfect absorber.

Numerical data are presented for a quartz receiver of the thickness type, and for a lengthwise-type receiver with crystals of ADP*. For the latter case curves representing the performance as a function of efficiency are given. (Contractor's abstract)

(See also TIP U8713)

* ammonium dihydrogen phosphate

TABLE OF CONTENTS

***
***

A GENERALIZED THEORY OF THE

CRYSTAL TRANSMITTER AND RECEIVER FOR PLANE WAVES

Symbols

$V' = Ve^{j\omega t}$ = instantaneous e.m.f. between electrodes

ξ_0 to ξ_7 = amplitudes of waves shown in Fig. 1

θ_0 = phase angle of ξ_0 with respect to V at $x = -\ell_b$

θ_1, θ_2 = phase angles of ξ_1 and ξ_2 with respect to V at $x = 0$

θ_3, θ_4 = phase angles of ξ_3 and ξ_4 with respect to V at $x = \ell$

θ_5, θ_6, θ_7 = phase angles of ξ_5, ξ_6 and ξ_7 with respect to V at $x = \ell + \ell_d$

$\rho_b c_b$, ρc, $\rho_d c_d$, and $\rho_o c_o$ = acoustic resistivities of backing, crystal, front plate, and liquid

$m_b = \rho_b c_b/\rho c$; $m_d = \rho_d c_d/\rho c$; $m = \rho_o c_o/\rho c$

$\lambda_b = c_b/f$, $\lambda = c/f$, $\lambda_d = c_d/f$, wavelengths in backing, crystal, and front plate

$\beta_b = \omega \ell_b/c_b = 2\pi \ell_b/\lambda_b$; $\beta = \omega \ell/c = 2\pi \ell/\lambda$; $\beta_d = \omega \ell_d/c_d = 2\pi \ell_d/\lambda_d$

$B_b = e^{-j\beta_b}$; $B = e^{-j\beta}$; $B_d = e^{-j\beta_d}$

$q_b = \rho_b c_b^2$, $q = \rho c^2$, $q_d = \rho_d c_d^2$, elastic stiffness-constants of backing, crystal, and front plate

S, T = strain and stress. Both are positive when extensional.

$y_n = \xi_n e^{j\theta_n}$ for n from 0 to 7

H = effective piezoelectric stress-constant

n = total number of crystal plates in the lengthwise transducer, each of width w and thickness t.

A = radiating area. In the lengthwise transducer, A = nwt.

ℓ = crystal dimension in the wave-direction. With thickness vibrations, ℓ is the thickness; with lengthwise vibrations, ℓ is the length.

N = $HV/\omega\ell\rho c$ (thickness type) or $HV/\omega t\rho c$ (lengthwise type)

ε^S = permittivity at constant strain, used with thickness vibrations

ε_ℓ = permittivity for lengthwise vibrations

ψ = $2H^2A/\ell^2\rho c$ (thickness type); $\psi = 2H^2A/t^2\rho c$ (lengthwise type)

ψ_1 = $2\pi mq^V\psi/H$ (thickness type); $\psi_1 = 2\pi mq^E t\psi/\ell H$ (lengthwise type)

INTRODUCTION

The transducer theory previously reported[1,2] has now been extended to include the case in which the transducer acts as a receiver of normally incident plane waves. The extension consists in the recognition of two sources of excitation, one electric, the other acoustic. When the electrical excitation alone is present, the load is acoustic and the device is a transmitter. When the excitation is acoustic, and the transducer terminals are connected to a passive electric network, there is also some electric excitation due to the reaction of the network.

Our problem is to assemble a set of equations based on boundary conditions, the solution of which will yield information on the transducer performance. The same equations are applicable whether the crystals are in lengthwise or thickness vibration, and whether the transducer acts as a transmitter or a receiver. The crystal assembly may have front and back plates of any conducting materials and thicknesses. Losses in these plates and in the crystals are ignored, but losses in the mounting are represented by an electrical equivalent.

---

1 W. G. Cady, "A Theory of the Crystal Transducer for Plane Waves," Technical Report No. 2, September 29, 1948, Contract N6onr-262, Wesleyan University; published in Jour. Acous. Soc. Am., 21, 65-73 (1949). This paper deals with crystals in lengthwise vibration. U8713

2 W. G. Cady, "Piezoelectric Equations of State and Their Application to Thickness-Vibration Transducers," Technical Report No. 7, March 20, 1950, Contract N6onr-262, Wesleyan University; published in Jour. Acous. Soc. Am., 22, 579-583 (1950). 6

The theory will be given for a transducer of the thickness-vibration type. Later it will be shown that by modifying the definitions of certain parameters the same equations can be used with the lengthwise type. As shown in Fig. 1, the crystal C, or mosaic of crystals, is cemented between a back plate B and a front plate (the "diaphragm") D. The total thickness, $\ell_b + \ell + \ell_d$, may be equivalent to a half wavelength, with ℓ relatively

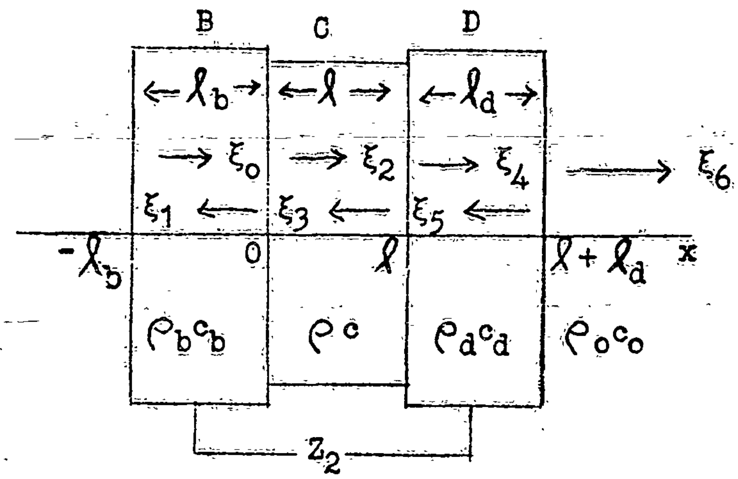


Fig. 1

Transducer consisting of crystal C, backing B, and front plate D

small, as in the Langevin quartz-steel sandwich, or the crystal itself may be a half-wavelength in thickness. In the general formulation no restriction is made with regard to dimensions, except that the radiating area A has dimensions large in comparison with the wavelength.

In the steady state the vibration in each component of the transducer is represented by two oppositely traveling waves. In the transmitter, ξ_6 is the amplitude of the emitted wave, and Z_2 is replaced by the impressed voltage V. There is no incident wave, so that $\xi_7 = 0$. In the receiver, ξ_7 is the incident and ξ_6 the reflected amplitude, the latter being dependent on the characteristics of the transducer and on the nature of the electrical load, Z_2.

The procedure is simplified by expressing all phase angles in terms of the phase of V for both transmitter and receiver, with $V' = Ve^{j\omega t}$.

The x-axis is parallel to the direction of wave propagation, with the origin at that crystal boundary which is more remote from the radiating medium. The metallic back and front plates B and D serve as electrodes for the output impedance Z_2 or for the driving generator. The surface of B at $-\ell_b$ is in contact with air, so that reflection is practically perfect. This fact makes it possible to express ξ_1 in terms of ξ_0, thereby reducing the number of variables by one.

By employing the concept of traveling waves[1,2] the boundary conditions can be formulated, leading to a set of seven simultaneous equations. In the receiver, the unknowns are ξ_0, ξ_2, ξ_3, ξ_4, ξ_5, ξ_6, and V, to be solved in terms of frequency, ξ_7, and Z_2.

It is assumed that all viscous and other mechanical losses in the transducer can be represented, at any particular frequency, by a resistance in parallel with Z_2.

Rationalized mks units will be used except when otherwise specified. Instantaneous values are denoted by prime accents.

EFFECT OF STRAIN-DISTRIBUTION ON THE ELASTIC CONSTANTS

OF PLATES VIBRATING IN A THICKNESS MODE

With crystals of relatively low coupling, like quartz, this effect is small, and especially so in the quartz-metal sandwich, where the thickness of the crystal layer is small in comparison with the wavelength. With crystals of strong coupling the effect may be far from negligible. In any case it is desirable to calculate its magnitude.

The following procedure is applicable to all thickness-type crystal transmitters and receivers. Since both the direction of wave-propagation and the electric field-direction are parallel to x, the problem is one-dimensional. The waves themselves are here assumed to be compressional. If they were transverse, all equations would be unaltered except for certain subscripts.

The appropriate equations of state are those giving the instantaneous stress and electric displacement in any small volume element.[3]

$$T_1'(x) = c_{11}^E S_1'(x) - e_{11} E_1'(x) \tag{1}$$

$$D_1'(x) = e_{11} S_1'(x) + \varepsilon^S E_1'(x) \tag{2}$$

For simplicity and to avoid confusion later, we omit the subscripts and write q^E for c_{11}^E, H for e_{11}, obtaining

$$T'(x) = q^E S'(x) - HE'(x) \tag{3}$$

$$D'(x) = HS'(x) + \varepsilon^S E'(x) \tag{4}$$

[3] Specialized from footnote 2, Eqs. (4) or (5).

In the vibrating plate the electric field has, as shown below, three terms, two due to polarization space charge, which is a consequence of the strain-distribution, and one to the potential difference V between the electrodes, the latter assumed to be in contact with the crystal. On the other hand the electric displacement is at all times uniform throughout the crystal, so that one may write D' for $D'(x)$. Then from Eq. (4), with $S'(x) = \partial\xi'(x)/\partial x$

$$V' = \int_0^{\ell} E'dx = \frac{1}{\varepsilon^S}\int_0^{\ell}\left(-H\frac{\partial\xi'(x)}{\partial x} + D'\right)dx = \frac{1}{\varepsilon^S}\left[-H\left\{\xi'(\ell) - \xi'(0)\right\} + D'\ell\right]$$

Therefore

$$D' = \frac{\varepsilon^S V'}{\ell} + \frac{H}{\ell}\left\{\xi'(\ell) - \xi'(0)\right\} \tag{5}$$

From Eqs. (4) and (5) we find for the total field[4]

$$E'(x) = -\frac{H}{\varepsilon^S}S'(x) + \frac{H}{\varepsilon^S\ell}\left\{\xi'(\ell) - \xi'(0)\right\} + \frac{V'}{\ell} \tag{6}$$

Of the three terms on the right in Eq. (6) the first is proportional to $S'(x)$ and contributes to the effective stiffness. The contribution of the other terms to the effective stiffness is discussed below.

---

[4] The first two terms on the right in Eq. (6) are due to the space charge induced by the space-variation of strain. See footnote 5; also W. G. Cady, Physics 6, 10-13 (1935).

From Eqs. (3) and (6) the total external mechanical stress on the volume element is

$$T'(x) = q^D S'(x) - \frac{H^2}{\varepsilon^S \ell}\left\{\xi'(\ell) - \xi'(0)\right\} - \frac{HV'}{\ell} \qquad (7)$$

where $q^D = q^E + H^2/\varepsilon^S$, the well-known stiffness at constant displacement. The second term on the right in Eq. (7) represents a uniform stress, independent of x and opposed to the deformation. At frequencies in the neighborhood of resonance it can be treated as a contribution to the effective stiffness. According to a method that has been described previously,[5] and by use of Eqs. (11) below, it can be proved that this contribution to the stiffness is $8H^2/\pi^2\varepsilon^S$. Eq. (7) is thereby converted to

$$T'(x) = q^D S'(x) - \frac{8H^2}{\pi^2\varepsilon^S} S'(x) - \frac{HV'}{\ell} \equiv q^V S'(x) - \frac{HV'}{\ell} \qquad (7a)$$

where $$q^V = q^D - \frac{8H^2}{\pi^2\varepsilon^S} \qquad (8)$$

The superscript V denotes the stiffness at constant V; that is, when V' is independent of $S'(x)$, so that $\partial T'(x)/\partial S'(x) = q^V$. This coefficient q^V is distinguished from q^E, the stiffness at constant field, by the fact that although, as in the transmitter, V is independent of the strain, this is by no means true of the field E.

The wave-velocity c is related to q^V by

$$q^V = \rho c^2 \qquad (9)$$

[5] W. G. Cady, "Piezoelectricity," McGraw-Hill Book Company, Inc., New York, 1946, pp. 312-316.

The stiffness q^V is to be used in the equations for the transmitter when there is no gap between electrodes and crystals. In the case of the receiver the voltage V in Eq. (7) is a function of the admittance Y_2 of the output circuit. V is no longer the driving voltage, but is rather to be treated as contributing still another term to the effective stiffness. As will be shown later, the effect is so small that it can usually be ignored. In the present discussion it is assumed that the effective stiffness is q^V for thickness transducers and q^E for lengthwise transducers.

BOUNDARY CONDITIONS

As has been proved in footnote 1, the assumption of perfect reflection at $x = -\ell_b$ leads to $y_o = B_b y_1$, while at $x = 0$ the equality of particle displacement gives Eq. (19a) below.

For expressing the equality of stresses at $x = 0$ and $x = \ell$ the following equations will be used:[1]

$$T'(0) = j\frac{2\pi q_b}{\lambda_b}(-B_b + \frac{1}{B_b})y_o e^{j\omega t} \qquad (11a)$$

$$T'(\ell) = j\frac{2\pi q_d}{\lambda_d}(-y_4 + B_d y_5)e^{j\omega t} \qquad (11b)$$

$$\xi'(0) = (y_2 + By_3)e^{j\omega t} \qquad (11c)$$

$$\xi'(\ell) = (By_2 + y_3)e^{j\omega t} \qquad (11d)$$

$$S'(0) = j\frac{2\pi}{\lambda}(-y_2 + By_3)e^{j\omega t} \qquad (11e)$$

$$S'(\ell) = j\frac{2\pi}{\lambda}(-By_2 + y_3)e^{j\omega t} \qquad (11f)$$

Equations (11a) and (11b) represent the stresses impressed on the crystal by the back and front plates respectively.

From Equations (11b) and (11c),

$$\xi'(\ell) - \xi'(0) = (B - 1)(y_2 - y_3)e^{j\omega t} \tag{12}$$

When Eqs. (11a), (11e) and (12) are substituted in (7), with $x = 0$, the desired boundary condition is obtained. In so doing, use is made of the following expressions: $q_b = \rho_b c_b^2$; $c_b = f\lambda_b$; $q^V = \rho c^2$; $c = f\lambda$; $\rho_b c_b/\rho c \equiv m_b$; and $\omega\ell\rho c = 2\pi\ell\rho c^2/\lambda = \beta q^V$. Then, on re-grouping, one finds

$$jm_b\left(-B_b + \frac{1}{B_b}\right)y_o + \left(j + \frac{H^2(B-1)}{\varepsilon^S\beta q^V}\right)y_2 - \left(jB + \frac{H^2(B-1)}{\varepsilon^S\beta q^V}\right)y_3 + N = 0 \tag{13}$$

where $N \equiv HV/\omega\ell\rho c = HV/\beta q^V$.

By the same procedure it is found that at $x = \ell$,

$$\left(jB + \frac{H^2(B-1)}{\varepsilon^S\beta q^V}\right)y_2 - \left(j + \frac{H^2(B-1)}{\varepsilon^S\beta q^V}\right)y_3 - jm_d(y_4 - B_d y_5) + N = 0 \tag{14}$$

The term $H^2(B-1)/\varepsilon^S\beta q^V$ is the correction due to space charge. If it is sufficiently small it may be dropped. For example, with a quartz x-cut plate, $H^2 = 0.030$, $\varepsilon^S = 3.9(10^{-11})$, $q^V = 8.83(10^{10})$, so that $H^2/\varepsilon^S q^V = 8.7(10^{-3})$. As to the magnitude of $(B-1)/\beta$, we have $B = e^{-j\beta} = \cos\beta - j\sin\beta$, where $\beta = 2\pi\ell/\lambda$. In the quartz-steel sandwich, $\ell << \lambda$; if $\ell = \lambda/10$, $(B-1)/\beta = -0.304 + 0.935j$, and the space-charge correction can be ignored unless high precision is required. At the other extreme, if $\ell = \lambda/2$, $\beta = \pi$, $B = -1$, and again the correction can be ignored.

The expressions for equality of particle displacements at $x = \ell$, and for equality of particle displacements and stresses at $x = \ell + \ell_d$, are Eqs. (19c), (19e), and (19f) below.[1]

The last of the equations needed for the solution of the problem is that for the current to the external circuit. In the receiver the current is due to the strain produced by the incident radiation. It is generated in the LC branch of the equivalent crystal network (which for the present purpose is most conveniently represented as LCC_1r) and is denoted by I_p in Fig. 2. The branch r is the electrical equivalent of all transducer losses, while Z_2 is the impedance of the external circuit. The parallel capacitance of the crystal is $C_1 = \varepsilon^S A/\ell$, where A is the area.

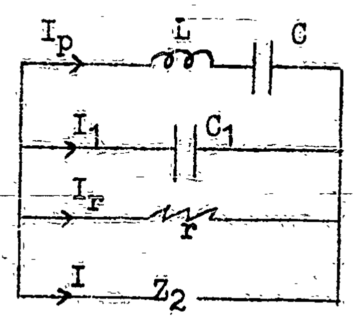


Fig. 2

Equivalent Circuit of the Crystal Receiver

From Kirchhoff's law,

$$I_p' + I_1' + I_r' + I' = 0 \qquad (15)$$

In this equation, $I_r' = V'/r = V'G_r$ (15a)

and $$I' = V'/Z_2 = V'Y_2 = V'(G_2 - jB_2) \qquad (15b)$$

The value of the first two terms in Eq. (15) is found by taking the time-derivative of Eq. (5). Then with the aid of Eq. (12), together with $V' = Ve^{j\omega t}$, we find for a transducer with active area A,

$$j\omega C_1 V + j\frac{\omega HA}{\ell}(B-1)(y_2-y_3) + G_r V + Y_2 V = 0$$

This expression is now multiplied by $H/\omega\ell\rho c$ and rearranged:

$$\frac{H^2A(B-1)}{\ell^2\rho c}(y_2-y_3) - jN(-jB_1 + G_r + Y_2) = 0 \tag{16}$$

where $B_1 = -\omega C_1$. The total admittance as seen from the LC branch is

$$Y_t = G_r + G_2 - j(B_1+B_2) = G_{2r} - j(B_1+B_2) = G_r + Y_{12} \tag{17a}$$

where $G_{2r} = G_r + G_2$, $Y_2 = G_2 - jB_2$, and $Y_{12} = Y_2 - jB_1 = G_2 - j(B_1+B_2)$ (17b)

Equation (16) can now be written as

$$\frac{H^2A(B-1)}{\ell^2\rho c}(y_2 - y_3) - jY_tN = 0 \tag{18}$$

From the foregoing, the system of simultaneous equations is as follows. Eq. (19b) comes from (13), (19d) from (14), and (19g) from (18).

GENERAL TRANSDUCER EQUATIONS

$$
\begin{array}{lllllllll}
(B_b + \frac{1}{B_b})y_o & -y_2 & -By_3 & +0 & +0 & +0 & +0 & = & 0 \qquad (19a) \\[2ex]
m_b(-B_b + \frac{1}{B_b})y_o & +(1-j\frac{H^2(B-1)}{\varepsilon^S\beta q^V})y_2 & -(B-j\frac{H^2(B-1)}{\varepsilon^S\beta q^V})y_3 & +0 & +0 & +0 & -jN & = & 0 \qquad (19b) \\[2ex]
0 & +By_2 & +y_3 & -y_4 & -B_d y_5 & +0 & +0 & = & 0 \qquad (19c) \\[2ex]
0 & +(B-j\frac{H^2(B-1)}{\varepsilon^S\beta q^V})y_2 & -(1-j\frac{H^2(B-1)}{\varepsilon^S\beta q^V})y_3 & -m_d y_4 & +m_d B_d y_5 & +0 & -jN & = & 0 \qquad (19d) \\[2ex]
0 & +0 & +0 & +m_d B_d y_4 & -m_d y_5 & -my_6 & +0 & = & -my_7 \qquad (19e) \\[2ex]
0 & +0 & +0 & +B_d y_4 & +y_5 & -y_6 & +0 & = & y_7 \qquad (19f) \\[2ex]
0 & +\frac{H^2A(B-1)}{\ell^2\rho c}y_2 & -\frac{H^2A(B-1)}{\ell^2\rho c}y_3 & +0 & +0 & +0 & -jY_tN & = & 0 \qquad (19g)
\end{array}
$$

In these equations there is no restriction on the material and thickness of the crystals and of the back and front plates, beyond the assumption that internal losses are negligible. Since the second terms in the coefficients of y_2 and y_3 in Eqs. (19b) and (19d) have been shown to be relatively small, they can usually be dropped. In many cases the back and front plates are of the same material and thickness, so that $m_b = m_d$ and $B_b = B_d$. When β is sufficiently small, approximation formulas for the trigonometrical functions can be used.

APPLICATIONS OF THE GENERAL TRANSDUCER EQUATIONS

Specialization Rules. When there is no back plate, $B_b = 1$, Eq. (19a) ~~drops out, also the first term in (19b).~~

When $\ell_b = \lambda_b/2$ (backing resonance), $B_b = -1$.

When there is no front plate, $B_d = 1$, and $m_d = m$.

As the crystal thickness ℓ approaches zero, B approaches 1.

When $\ell = \lambda/2$ (crystal resonance), $B = -1$.

When $\ell_d = \lambda_d/2$ (diaphragm resonance), $B_d = -1$.

When the transducer acts as a transmitter with a given impressed voltage V, $y_7 = 0$, Eq. (19g) drops out, and N is treated as a known quantity.

Application of these rules will now be made to some practical cases.

I. Thickness-type Transmitter.

With $y_7 = 0$ and Eq. (19g) out, the number of equations becomes reduced to six. If there is no back plate, and furthermore if y_6 is eliminated between Eqs. (19e) and (19f), four equations remain. With the space-charge correction omitted the equations are:

$$
\begin{array}{llllcll}
 & & y_2 & -By_3 & = & jN & \quad (20a) \\
By_2 & +y_3 & -y_4 & -B_d y_5 & = & 0 & \quad (22b) \\
By_2 & -y_3 & -m_d y_4 & +m_d B_d y_5 & = & jN & \quad (22c) \\
 & & (m-m_d)B_d y_4 & +(m+m_d)y_5 & = & 0 & \quad (22d)
\end{array}
$$

This case is treated in footnote 2, where it is also shown that the proper elastic stiffness-coefficient is $q^V = q^D - 8H^2/\pi^2\varepsilon^S$. See also footnote 5 above.

For a crystal plate radiating directly into a medium $\rho_o c_o = m\rho c$, with air backing, Eqs. (20) can be reduced to the form[6]

$$y_2 \qquad -By_3 = jN \qquad (21a)$$

$$(1-m)By_2 \quad -j(1+m)y_3 = jN \qquad (21b)$$

The results derived from Eqs. (20) and (21) are discussed in the papers cited, and need not be repeated here.

II. Thickness-type Receiver with front plate but no back plate.

The only case considered here is that in which the transducer is in resonance with the incident radiation, so that $\ell = \lambda/2$, $\beta = \pi$, and $B = -1$. The front plate may have any thickness. Equations (19) become reduced to

$$\left(1+j\frac{2H^2}{\pi\varepsilon S_q V}\right)y_2 \quad +\left(1-j\frac{2H^2}{\pi\varepsilon S_q V}\right)y_3 \qquad -jN = 0 \qquad (22a)$$

$$y_2 - y_3 + y_4 + B_d y_5 = 0 \qquad (22b)$$

$$\left(1-j\frac{2H^2}{\pi\varepsilon S_q V}\right)y_2 \quad +\left(1+j\frac{2H^2}{\pi\varepsilon S_q V}\right)y_3 + m_d y_4 - m_d B_d y_5 + jN = 0 \qquad (22c)$$

$$m_d B_d y_4 - m_d y_5 - m y_6 = -m y_7 \qquad (22d)$$

$$B_d y_4 \quad + y_5 \quad - y_6 = y_7 \qquad (22e)$$

$$\frac{2H^2 A}{\ell^2 \rho c} y_2 \quad - \frac{2H^2 A}{\ell^2 \rho c} y_3 \qquad + jY_t N = 0 \qquad (22f)$$

---

6 Report WUX-1, Sub-contract DIC-178188 between the Radiation Laboratory, Massachusetts Institute of Technology, and Wesleyan University, September 30, 1945.

As before, the imaginary terms in the coefficients of y_2 and y_3 can usually be dropped.

As a further special case it is assumed that these imaginary terms can be dropped, and also that the front plate has a thickness $\ell_d = \lambda_d/2$, so that $B_d = -1$. Then from Eqs. (22) one finds

$$y_2 + y_3 - jN = 0 \tag{23a}$$

$$y_2 - y_3 + y_4 - y_5 = 0 \tag{23b}$$

$$y_2 + y_3 + m_d y_4 + m_d y_5 + jN = 0 \tag{23c}$$

$$m_d y_4 + m_d y_5 + m y_6 = m y_7 \tag{23d}$$

$$-y_4 + y_5 - y_6 = y_7 \tag{23e}$$

$$\frac{2H^2A}{\ell^2 \rho c} y_2 - \frac{2H^2A}{\ell^2 \rho c} y_3 + jY_t N = 0 \tag{23f}$$

The number of equations can be reduced to five if solutions for y_2 and y_3 separately are not needed. We therefore write

$$y_2 + y_3 \equiv u \tag{24}$$

also, for brevity,

$$\frac{2H^2A}{\ell^2 \rho c} \equiv \psi \tag{25}$$

Then after a little manipulating we obtain

$$u - jN = 0 \qquad (26a)$$

$$\frac{Y_t}{\psi}u - y_4 + y_5 = 0 \qquad (26b)$$

$$u + m_d y_4 + m_d y_5 + jN = 0 \qquad (26c)$$

$$-y_4 + y_5 - y_6 = y_7 \qquad (26d)$$

$$m_d y_4 + m_d y_5 + m y_6 = m y_7 \qquad (26e)$$

Before proceeding to the solution of these equations we will show that they, as also Eqs. (23), hold also for receiving transducers of the lengthwise type.

III. Lengthwise-type Receiver.

In this type of transducer the electric field is at right angles to the direction of wave propagation. It is assumed that the length ℓ of the individual crystal unit is in the x-direction, while the thickness t, and therefore the electric field, is in the z-direction. The electric polarization is variable in the x-direction but not in the z-direction. Therefore there is no polarization space charge in the field direction, and we have for the instantaneous electric field only $E' = V'/t$. The stiffness-constant is $q^E = 1/s_{11}^E$ (the small correction due to the output load is treated later). The effective piezoelectric constant is $\bar{h} = d_{31}/s_{11}^E = d_{31}q^E$, as stated in footnote 1. On the assumption that there are n crystal plates, each of length ℓ, width w, and thickness t, the radiating area is $A = nwt$. In practice w may be of the same order of magnitude as ℓ. The parameters N and ψ are now defined as

$$N = \frac{HV}{\omega t \rho c} \qquad \psi = \frac{2H^2 A}{t^2 \rho c} \tag{27}$$

where $c = (q^E/\rho)^{\frac{1}{2}}$.

To convert Eqs. (19) into the proper form for lengthwise-type receivers it is only necessary to drop the second term in the coefficients of y_2 and y_3 in Eqs. (19b) and (19d), and to substitute t^2 for ℓ^2 in Eq. (19g).

In order to indicate how this comes about, it is enough to give independent derivations of Eqs. (23c) and (23f) for the lengthwise case. Eq. (23c) expresses the equality of stresses at $x = \ell$, and is based on the equation of state

$$S_1'(x) = s_{11}^E T_1'(x) + d_{31} E_3'(x) \tag{28}$$

Upon setting $x = \ell$, $E_3'(x) = V'/t = Ve^{j\omega t}/t$, $S_1'(\ell) = j(2\pi/\lambda)(-By_2+y_3)e^{j\omega t}$, $s_{11}^E = 1/q^E$, and then solving for $T_1'(\ell)$, we find

$$T_1'(\ell) = j\frac{2\pi q^E}{\lambda}(-By_2 + y_3)e^{j\omega t} - \frac{d_{31}q^E V}{t}e^{j\omega t} \tag{29}$$

$T_1'(\ell)$ is the negative of the pressure of the front plate against the crystal, and is to be equated to Eq. (11b). At resonance, $B_d = B = -1$. Then, with $d_{31}q^E = H$, Eq. (29) becomes

$$-j\frac{2\pi q_d}{\lambda_d}(y_4 + y_5) = j\frac{2\pi q^E}{\lambda}(y_2 + y_3) - \frac{HV}{t}$$

We now divide both sides by $\omega\rho c$ and note that $2\pi q_d/\lambda_d\omega\rho c$ = $\rho_d c_d/\rho c$ = m_d, also that $2\pi c^E/\lambda\omega\rho c = 1$, thus obtaining

$$-m_d(y_4 + y_5) = u + j\frac{HV}{\omega\ell\rho c} = u + jN,$$

in agreement with Eqs. (23c) and (26c). The proof for Eq. (26a), at $x = 0$, is similar.

In deriving the equation for current we use Eqs. (15) to (15b). In expressing the electric displacement, however, Eq. (5) is not to be used, but rather the equation of state in terms of stress:

$$D_3'(x) = d_{31}T_1'(x) + \varepsilon^T E_3' \tag{30}$$

When $T_1'(x)$ is eliminated between Eqs. (30) and (28), there results

$$D_3'(x) = HS_1'(x) + \varepsilon_\ell E_3'$$

where $\varepsilon_\ell = \varepsilon^T - d_{31}^2/s_{11}^E$ and $S_1'(x) = j\frac{2\pi}{\lambda}(-y_2 e^{-j\frac{2\pi x}{\lambda}} - y_3 e^{j\frac{2\pi x}{\lambda}})e^{j\omega t}$ from footnote 1. The current through the crystal (see Fig. 2) is

$$I_p' + I_1' = nw\frac{\partial}{\partial t}\int_0^\ell D_3'(x)dx = j\omega nw\left\{\frac{H\lambda}{\ell}(-y_2 + y_3) + \varepsilon_\ell \ell E_3\right\}e^{j\omega t} \tag{31}$$

Upon writing $nwt = A$, $2H^2A/t^2\rho c = \psi$, $\varepsilon_\ell A/t = C_1$ and $HV/\omega t\rho c = N$; and combining Eqs. (15), (15a), (15b), (17a), and (31), one arrives at Eq. (23f). Equations (23b), (23d), and (23e) are all valid for the lengthwise-type receiver. Therefore Eqs. (23) and (26) hold for receivers of both thickness and lengthwise types.

THE TUNED CRYSTAL RECEIVER

The rest of this paper is concerned with the transducer designed for receiving at a single frequency, having crystals with $\ell = \lambda/2$, with or without a front plate for which $\ell_d = \lambda_d/2$. If the front plate has no internal losses it has no effect on the results at resonance, although it does increase the quality factor Q and thereby makes the transducer slightly more sensitive to changes in frequency. Similarly, if there were a half-wavelength back plate it would increase Q still further without affecting the solution at resonance.

The solution is to be derived from Eqs. (26). It has been shown that these equations hold for transducers of either the lengthwise or the thickness type, if the proper definitions are attached to N and ψ.

The solution of Eqs. (23) or (26) yields the following relations, in which Y_t is expressed according to Eq. (17a):

$$y_6 = \frac{(m^2G^2_{2r} - 4\psi^2) + m^2(B_1 + B_2)^2 - j4m\psi(B_1 + B_2)}{(mG_{2r} + 2\psi)^2 + m^2(B_1 + B_2)^2} y_7 \qquad (32)$$

$$V = -\psi_1 \frac{m(B_1 + B_2) - j(mG_{2r} + 2\psi)}{(mG_{2r} + 2\psi)^2 + m^2(B_1 + B_2)^2} y_7 \qquad (33)$$

where $$\psi_1 = \frac{2\pi m \psi q^E t}{\ell H} \qquad (33a)$$

for the lengthwise type, and

$$\psi_1 = \frac{2\pi m \psi q^V}{H} \qquad (33b)$$

for the thickness type.

The magnitude and phase of V with respect to ξ_7 are found by setting $\bar{y}_7 = \xi_7 e^{j\theta_7} = \xi_7(\cos\theta_7 + j\,\sin\theta_7)$ in Eq. (33) and equating real and imaginary parts:

$$V = -\psi_1 \frac{m(B_1 + B_2)\cos\theta_7 + (mG_{2r} + 2\psi)\sin\theta_7}{(mG_{2r} + 2\psi)^2 + m^2(B_1 + B_2)^2}\xi_7 \qquad (34a)$$

$$\tan\theta_7 = \frac{mG_{2r} + 2\psi}{m(B_1 + B_2)} \qquad (34b)$$

When $B_2 = -B_1$ these expressions become

$$V = \frac{\psi_1\xi_7}{mG_{2r} + 2\psi} \qquad (34c)$$

$$\tan\theta_7 = \infty, \qquad \theta_7 = 90^\circ \qquad (34d)$$

Thus when $B_2 = -B_1$, V lags 90° behind ξ_7.

The dependence of V upon H and the electrical admittance merits a short discussion. The significance will not be lessened if for simplicity, and in conformity with common practice, we set $B_2 = -B_1$, so that Y_t is a pure conductance. For the lengthwise type Eq. (34c) becomes

$$V = -\frac{4\pi m q^E t A}{\ell} \frac{H}{t^2\rho_o c_o G_{2r} + 4AH^2}\xi_7 \qquad (35)$$

When $G_{2r} = \infty$, the crystal is short-circuited and $V = 0$ whatever H may be. As G_{2r} decreases, V increases uniformly until $G_2 = 0$ and only the loss conductance G_r remains; V then has its greatest possible value with given G_r.

If even G_r were absent we would have the ideal transducer, with

$$V = -\frac{\pi m q^E t}{\ell H}\xi_7 = -\frac{\omega \rho_o c_o t}{H}\xi_7 \tag{36}$$

Next consider the relation of V to H. For a given G_{2r}, V in Eq. (35) has a maximum when

$$H^2 = \frac{mt^2 \rho c}{4A} G_{2r} = \frac{\rho_o c_o t^2}{4A} G_{2r} \equiv H_m^2 \tag{37}$$

where H_m is the value of H that makes V a maximum for a given G_{2r}. From Eq. (35), with $m = \rho_o c_o / \rho c$ and $q^E = \rho c^2 = 2f\ell\rho c$, the corresponding maximal V is

$$V_{max} = -\omega\xi_7 \sqrt{\frac{\rho_o c_o A}{G_{2r}}} = -\frac{\omega \rho_o c_o t}{2H_m}\xi_7 \tag{38}$$

For values of H smaller or larger than that given by Eq. (37), V decreases, approaching zero as H approaches zero.

V_{max} is obviously greatest when $G_2 = 0$ and G_{2r} is made as small as possible, and when the piezoelectric constant H has the value given by Eq. (37). ~~It is a curious circumstance, and at first sight an apparently paradoxical one, that as a generator of voltage a receiving transducer should have crystals of low rather than high piezoelectric constant. The explanation is that in the present case the transducer is not a generator of electrical power. The vibrational amplitude is proportional to the input acoustic power. The electric displacement, and therefore the current I_p in Fig. 2, are proportional to this amplitude and to H. When H is large, I_p is large, and since~~

~~$V = I_p/G_{2r}$ it is clear from Eq. (38) that V_{max} varies inversely with I_p and therefore inversely with H.~~

The same conclusions apply also to receivers of the thickness type.

Since V is the peak voltage, the power supplied to the external circuit is

$$P = \frac{1}{2} V^2 G_2 = \frac{\Psi_1^2}{2} \frac{G_2}{(mG_2 + mG_r + 2\Psi)^2 + m^2(B_1 + B_2)^2} \xi_7^2 \tag{39}$$

Equation (39) shows that the power is always increased by making $B_2 = -B_1$, which means compensating for C_1 by a suitable inductance.

The total power absorbed from the incident radiation, including losses, is

$$P_t = \frac{1}{2} V^2 G_{2r} = \frac{\Psi_1^2}{2} \frac{G_2 + G_r}{(mG_2 + mG_r + 2\Psi)^2 + m^2(B_1 + B_2)^2} \xi_7^2 \tag{40}$$

This expression can also be derived by making use of the fact that all of the incident energy that is not absorbed by the transducer goes into the reflected wave, so that $P_t = \frac{1}{2}\omega^2 A \rho_o c_o (\xi_7^2 - \xi_6^2)$. ξ_7 and ξ_6 are the moduli of y_7 and y_6, and y_6 is given by Eq. (32).

It is of interest to determine the value of the output electrical admittance that makes the useful power a maximum for given ξ_7. The derivatives of P with respect to G_2 and B_2 are set separately equal to zero, giving two simultaneous equations:

$$G_2^{\,2} = (G_r + 2\psi/m)^2 + (B_1 + B_2)^2$$

$$G_2(B_1 + B_2) = 0$$

When these equations are solved for G_2 and B_2, one finds (excluding the solution $G_2 = 0$ or negative),

$$\boxed{B_2 = -B_1} \qquad (41)$$

$$\boxed{G_2 = G_r + \frac{2\psi}{m} = G_r + \frac{4H^2A}{t^2\rho_o c_o}} \qquad (42)$$

Using these values we find from Eq. (39), for maximal useful power,

$$P_{max} = \frac{\psi_1^{\,2}}{8m}\,\frac{1}{mG_r + 2\psi}\,\xi_7^{\,2} = \frac{2\pi^2 mq^{E^2}A^2}{\ell^2\rho c}\,\frac{H^2}{t^2\rho_o c_o G_r + 4H^2A}\,\xi_7^{\,2} \qquad (43)$$

The loss-conductance G_r can be measured, and ψ can be calculated from Eq. (25) or (27), so that from (42) the conductance G_2 for maximal output power can be found. In order to consume maximal power, the output circuit should have this conductance, together with a susceptance $B_2 = -B_1$.

The voltage V_p when P is a maximum is found, for lengthwise vibrations, from Eqs. (35) and (42):

$$V_p = -\frac{2\pi mq^E tA}{\ell}\,\frac{H}{t^2\rho_o c_o G_r + 4AH^2}\,\xi_7 \qquad (44)$$

A similar expression can be derived for thickness vibrations.

It is seen from Eqs. (35) and (44) that, when $B_2 = -B_1$, the voltage for $G_2 = 0$ is just twice as great as when G_2 has the value that makes the power a maximum.

If there are no losses, $G_r = 0$, and from Eqs. (41) and (42) the conditions for maximal power become $B_2 = - B_1$ and $G_2 = 2\psi/m$. When these substitutions are made in Eq. (32), it is found that $y_6 = 0$. This means that the ideal transducer can become a perfect absorber, no energy being reflected. The voltage V generated by the received energy makes the transducer act as a transmitter, emitting waves of amplitude $\xi_6 = - \xi_7$. How closely the actual transducer approaches the ideal depends on the losses; that is, on the transducer efficiency. For a given G_r, Eq. (43) shows that P_{max} increases with H. Therefore, in order to convert as much as possible of the accustic energy into useful power output, the transducer should contain crystals of high H.

EFFECT OF OUTPUT IMPEDANCE ON THE CRYSTAL VIBRATIONS

When plane waves of amplitude ξ_7 fall at normal incidence on a half-wave slab of isotropic no-loss solid, the faces of the solid vibrate with amplitude $2\xi_7$. There is a loop of motion and a node of strain at each face, while at the center of the slab there is a node of motion and a loop of strain. Reflection is complete. The reflected wave, with amplitude $\xi_6 = \xi_7$, is in phase with ξ_7, just as when the acoustic wave in air in a tube is reflected at the free end.

The effects described above remain unchanged when the half-wave slab is piezoelectric, with short-circuited electrodes ($Y_t = \infty$). The material still behaves as if isotropic. Let us suppose next that the electrodes are connected to a local oscillator of the same frequency as the acoustic input, but with controllable voltage and phase. A half-wave no-loss back or front plate, or both, may be present without affecting the results. By variation of voltage and phase the radiation emitted by the crystal can be made to have any value, greater or smaller than ξ_7, and in any phase relation to ξ_7. The applied voltage V causes a resonant vibration of amplitude ξ_V which is superposed on the vibration due to ξ_7. In particular, if V is such as to make the vector $\xi_V = -\xi_7$, the crystal has a resultant amplitude equal to and in phase with ξ_7 (since ξ_7 alone would produce the amplitude $2\xi_7$), and there is no radiation back into the medium. All the acoustic energy is then absorbed. On the other hand, if $\xi_V = -2\xi_7$, there is an emitted wave of amplitude $\xi_6 = -\xi_7$, and no motion at the boundary. Reflection is complete, but takes place as in air at the end of a closed tube, where the reflection is effectively from an infinitely stiff medium.

It will now be shown that in the ideal no-loss crystal receiver values can be assigned to the passive output admittance that will cause V to be exactly what is needed to produce the effects that have just been described. For this purpose we require the expression for ξ_V, the amplitude at ℓ due to V alone, in terms of V. It is given in footnote 1, Eq. (35), which for the resonant condition becomes simplified to $\xi(\ell) = \xi_d = 2N/m$. Considering only the lengthwise-type receiver, we have $N = HV/\omega t \rho c$, so that

$$\xi_V(\ell) = \frac{2HV}{\omega t \rho cm} = \frac{2HV}{\omega t \rho_o c_o} \tag{45}$$

For the condition of maximal power output, V is given by Eq. (34a) with $G_{2r} = G_2 + G_r = 2G_r + 2\psi/m$ from Eq. (42), so that (45) becomes reduced to

$$\xi_V(\ell) = \frac{-\xi_7}{1 + \dfrac{t^2 \rho_o c_o}{4AH^2} G_r} \tag{46}$$

When $G_r = 0$, $\xi_V(\ell) = -\xi_7$. The negative sign means a difference of 180° in phase. $\xi(\ell)$ lags 90° behind V, and, as we have seen, V lags 90° behind ξ_7. No energy is reflected, all being absorbed in the output circuit. The realizable approximation to this ideal condition depends on how small G_r can be made.

For the condition of perfect reflection we set $B_2 = -B_1$ and $G_{2r} = 0$. Then (ℓ) turns out to be equal to $-2\xi_7$, and there is a returning radiation of amplitude $\xi_6 = \xi_7$, with no motion at the crystal boundaries. That there is no motion at the boundaries can also be verified by solving Eqs. (23) for y_2

and y_3, and setting these values in the general equation for particle-displacement at resonance (fn. 1, Eq. (6)),

$$\xi'(x) = \left(y_2 e^{-j\frac{2\pi x}{\lambda}} - y_3 e^{j\frac{2\pi x}{\lambda}} \right) e^{j\omega t}$$

It is found thus that when $B_2 = -B_1$ and $G_{2r} = 0$, $\xi(0) = \xi(\ell) = 0$, while $\xi(\frac{\ell}{2}) = m\xi_7$. The crystal vibrates like a bar in resonant vibration with both ends clamped and a loop of motion at the center.

TRANSDUCER EFFICIENCY

A. Transmitter.

The input power is $P_i = \frac{1}{2} V^2 G_t$, where V is the peak voltage, and the total electrical conductance is $G_t = G + G_r$. $G_r = 1/r$ is the loss conductance (Fig. 2), and $G = 1/R$, where R is the resistance in the usual $RLCC_1$ crystal network. If crystal losses are ignored, R is due solely to the radiation resistance of the irradiated medium. For a thickness-type transducer consisting of n plates in lengthwise vibration, $R = p \ell t\alpha/4H^2 wn$; for the thickness type, $R = p\ell^3 \alpha/4H^2 A$, where ℓ is the thickness dimension. In either case the damping factor is $\alpha = cm/\ell = c \rho_0 c_0/\rho c \ell$.[6] With these data it is easily proved that in either case $G = 2\psi/m$.

The useful output is $P = \frac{1}{2} V^2 G$. The efficiency η is therefore

$$\eta = \frac{P}{P_i} = \frac{G}{G + G_r} = \frac{1}{1 + G_r/G} = \frac{1}{1 + mG_r/2\psi} \tag{47}$$

B. Receiver.

The input power is $P_i = \frac{1}{2} \omega \rho_0 c_0 A \xi_7^2$. On combining this with Eq. (39) one finds

$$\eta = \frac{P}{P_i} = \frac{8\psi m G_2}{(mG_2 + mG_r + 2\psi)^2 + m^2(B_1 + B_2)^2} \tag{48}$$

When for G_2 and B_2 the values given in Eqs. (41) and (42) are used, we find for the efficiency at maximal P

$$\eta = \frac{2\psi}{mG_r + 2\psi} = \frac{1}{1 + mG_r/2\psi} \tag{49}$$

This expression is the same as that for the transmitter in Eq. (47).

EFFECT OF OUTPUT ADMITTANCE

ON THE EFFECTIVE STIFFNESS OF THE RECEIVER

In the foregoing theory it was assumed that the wave-velocity c and the stiffness $q = \rho c^2$ were independent of the output. For both transmitter and receiver we used q^V and q^E for the thickness, and lengthwise types, respectively.

A small correction to these values must now be considered. In both types of receiver the voltage V depends in magnitude and phase on the output admittance. The resulting contribution to the field, V/ℓ, causes a stress which in turn affects the effective stiffness, as may be seen from Eq. (7a). Since this field is uniform throughout the crystal, the resulting stress is uniform and therefore not proportional to the strain; nor, in general, is it in phase with the strain. Nevertheless its effect on the effective stiffness can be calculated by a procedure analogous to that mentioned in footnote 5. For the lengthwise-type receiver the corrected effective stiffness is

$$q = q^E - \frac{8AH^2c}{\pi t^2} \frac{(B_1 + B_2) - jG_{2r}}{G_{2r}^2 + (B_1 + B_2)^2} \tag{50}$$

The corresponding expression for the thickness-type receiver is

$$q = q^V - \frac{8AH^2c}{\pi \ell^2} \frac{(B_1 + B_2) - jG_{2r}}{G_{2r}^2 + (B_1 + B_2)^2} \tag{51}$$

The complex character of the correction is due to the phase relation between V' and $T'(x)$ in Eq. (7a).

In calculating the corrected values of the velocity and resonant frequency from the relation $q = \rho c^2 = 4\rho f^2 \ell^2$, only the real part of q in Eqs. (50) and (51) is to be used. Thus for the lengthwise-type receiver,

$$\mathrm{Re}\, q = q^E - \frac{8AH^2 c}{\pi t^2} \frac{B_1 + B_2}{G_{2r}^2 + (B_1 + B_2)^2} \tag{52}$$

For the thickness-type receiver,

$$\mathrm{Re}\, q = q^V - \frac{8AH^2 c}{\pi \ell^2} \frac{B_1 + B_2}{G_{2r}^2 + (B_1 + B_2)^2} \tag{53}$$

As an illustration one may consider the case of the thickness-type receiver an open circuit, for which $G_2 = 0$ and $B_2 = 0$. If the receiver has no loss, $G_r = 0$, and since $B_1 = -\omega C_1 = -2\pi f \varepsilon^S A/\ell$, Eq. (53) becomes reduced to

$$\mathrm{Re}\, q = q^V + \frac{8H^2}{\pi^2 \varepsilon^S} = q^D \tag{54}$$

in accordance with Eq. (8). A similar relation can be proved for Eq. (52).

Whenever $B_2 = -B_1$, the correction vanishes, and the effective stiffness is simply q^E or q^V.

SUMMARY OF EFFECTS

OF OUTPUT CIRCUIT ON RECEIVER PERFORMANCE

I. $G_2 = \infty$ or $B_2 = \infty$. Short circuit, $V = 0$, $P = 0$. Use q^E for lengthwise, q^V for thickness type.

II. $G_2 = 0$, $B_2 = 0$. Open circuit, $P = 0$. Use q^D for both types of receiver.

III. $G_2 = 0$, $B_2 = -B_1$. C_1 is neutralized, $P = 0$. All incident energy that is not reflected is expended in overcoming losses in the transducer. For a given ξ_7, V has its greatest possible value, denoted by V_0, decreasing as G_r increases. If $G_r = 0$ all energy is reflected as from an immovable wall, and Eqs. (52) and (53) cease to have meaning.

IV. $B_2 = -B_1$, $G_2 > 0$. C_1 is neutralized, and useful power appears in the output. In the particular case where G_2 and G_r have specified values and H has the value H_m given by Eq. (37), V has its maximal value V_{max} with respect to H, for given ξ_7. For all values of G_r, as long as $G_2 + G_r = G_{2r}$ remains constant, and H_m remains constant, V_{max} remains unchanged. The stiffness q is given by Eq. (52) or (53).

V. $B_2 = -B_1$, $G_2 = G_r + 2\psi/m$ from Eq. (42). This is the condition for maximal P. V_P and P are greatest when $G_r = 0$, diminishing as G_r becomes greater. At all values of G_2, V_P has half the value of V_0 mentioned in II. The stiffness q is given by Eq. (52) or (53).

EXAMPLES OF RECEIVER CALCULATIONS

Using the best available numerical data, we have calculated various parameters for two hypothetical transducers, for different values of the loss-conductance G_L. Only the resonant frequency is considered, at which $\ell = \lambda/2$ for the crystals. As a first-order approximation the variation of stiffness with admittance of load, and consequently the small variation of resonant frequency, can be ignored. If no-loss half-wave front and back plates are present they do not affect the results.

I. Lengthwise type, consisting of ammonium dihydrogen phosphate (ADP) Z-cut 45° plates forming an assemblage with receiving area $A = 8 \times 8$ cm^2. The plates have dimensions $\ell = 0.041$ meters, $w = 0.02$ meters, $t = 0.005$ meters. The number of plates is $n = 64$, area $A = nwt = 6.4(10^{-3})$meter^2, $H = 0.473$ coulomb/meter^2, $q^E = 1.91(10^{10})$newton/meter^2, frequency $f = 40$ kc, $\rho = 1.804(10^3)$kg/meter^3, $c = 3.26(10^3)$meter/sec, $\rho c = 5.88(10^6)$kg meter^{-2} sec^{-1}, $\rho_o c_o = 1.55(10^6)$kg meter^{-2}sec^{-1} for sea water, $m = \rho_o c_o/\rho c = 0.263$, $\varepsilon_\ell = 1.27(10^{-10})$farad/meter. From these constants are calculated $\psi = 1.95(10^{-5})$, $\psi_1 = 1.60(10^5)$.

q^D is about 7 percent greater than q^E. For the curves in Fig. 3 only q^E is needed.

II. Thickness type, circular X-cut quartz plate resonating at 15 Mc. $A = 7.4(10^{-5})$meter^2, thickness $\ell = 1.92(10^{-4})$meter, $H = 0.173$ coulomb/meter^2, $q^V = 8.8(10^{10})$newton/meter^2, $\rho = 2.65(10^3)$kg/meter^3, $c = 5.75(10^3)$meter/sec, $\rho c = 15.2(10^6)$kg meter^{-2}sec^{-1}, $\rho_o c_o = 1.55(10^6)$, $m = \rho_o c_o/\rho c = 0.10$, $\varepsilon^S = 3.91(10^{-11})$ farad/meter. From these constants are calculated

$\psi = 7.8(10^{-6})$, $\psi_1 = 2.50(10^{6})$.

q^D is about 0.7 percent greater than q^V.

The theoretical performance of the lengthwise receiver is shown in Fig. 3. $G_r = 1/r$ is the equivalent conductance due to losses in the transducer; it is related to the efficiency η by Eq. (49). V_o is calculated from Eq. (34c) for $G_2 = 0$, $B_2 = -B_1$. V_{max} is found from Eq. (38), and the corresponding G_{2r} (not shown in Fig. 3) from Eq. (37). G_2 for V_{max} is $G_{2r} - G_r$. P for V_{max} comes from Eq. (39).

For the curves relating to maximal power, G_2 is calculated from Eq. (42), while $G_{2r} = G_2 + G_r$. P_{max} is found from Eq. (43), and the corresponding V_P from Eq. (44). In Fig. 3 the linear increase of both P_{max} and V_P with efficiency is made evident, together with the value of G_2 needed to make P a maximum.

Fig. 3 also illustrates the fact that at all efficiencies V_o is twice as great as V_P; but since V_o is the voltage when $G_2 = 0$, there is then no useful power.

On the other hand, when G_2 has the value needed to make V a maximum, then as long as the efficiency is above 50 percent V_{max} has a constant value equal to the value of V_P when $\eta = 1$. Useful power is then delivered, but it is equal to P_{max} only when $\eta = 1$. As the efficiency decreases, the power corresponding to V_{max} diminishes rapidly, becoming zero when $\eta = 0.5$.

For the thickness receiver the curves would have the same general form. It must suffice here to state that for an efficiency of 50 percent, $G_r = 15.6(10^{-5})$ mho and $r = 6400$ ohms. For maximal power, at 100 percent efficiency, G_2 would be $15.6(10^{-5})$, increasing as the efficiency decreased.

Fig. 3

Theoretical Curves for an ADP Receiver

Abscissas are values of the efficiency η when the output conductance G_2 is adjusted for maximal power P. The numbers on the vertical scale are to be multiplied by the following factors:

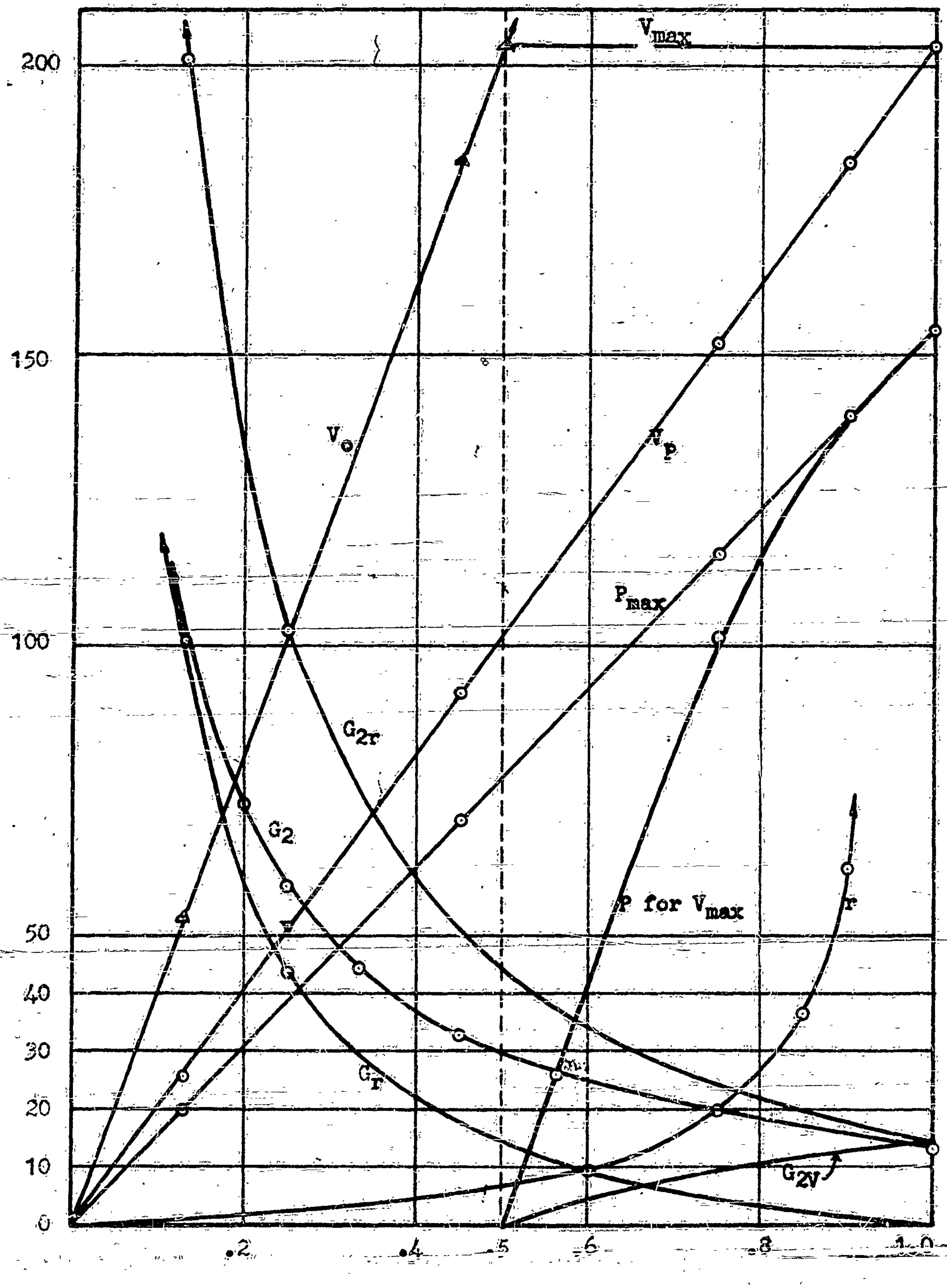
Conductances G in mhos, 10^{-5}

Resistance r in ohms, 10^3

V/ξ_7 in volts per meter, 10^7

$P/\xi_7{}^2$ in watts per meter^2, $2(10^{12})$

G_{2r} and G_2 are the values for max. P. G_{2V} is the value of G_2 for max. V. Circles and triangles are points for which values were calculated.



V_{max}
200
150
100
50
40
30
20
10
0
V_o
N_p
P_{max}
G_{2r}
G_2
P for V_{max}
r
G_r
G_{2V}
.2
.4
.5
.6
.8
1.0

Fig. 3

TECHNICAL REPORTS DISTRIBUTION LIST

WESLEYAN UNIVERSITY
CONTRACT N6onr-262, Task Order I
NR 014 401

A. Government Distribution

The National Military Establishment

Director,
Research and Development Board
Pentagon Building
Washington 25, D. C. (2 copies)

Department of the Navy

Chief of Naval Research
Office of Naval Research
Washington 25, D. C.
Attn: Physics Branch (2 copies)

ONR: Branch Offices

Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
495 Summer Street
Boston 10, Mass. (1 copy)

Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
346 Broadway
New York 13, N. Y. (1 copy)

Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
American Fore Building
844 N. Rush Street
Chicago 11, Illinois (1 copy)

Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
801 Donahue Street
San Francisco 24, California (1 copy)

Commanding Officer
U.S. Navy Office of Naval Research
Branch Office
1030 E. Greene Street
Pasadena 1, California (1 copy)

Office of the Assistant Naval Attache for Research
Navy 100
Fleet Post Office
New York, New York (2 copies)

Director,
U.S. Navy Underwater Sound Reference Laboratory
Office of Naval Research
P.O. Box 3629
Orlando, Florida (1 copy)

Director,
Naval Research Laboratory
Anacostia Station
Washington 20, D. C.
Attn: Sound Division (1 copy)

Director,
Naval Research Laboratory
Washington 20, D. C.
Attn: Technical Information Officer (9 copies)

Director,
U.S. Navy Electronics Laboratory
San Diego 52, California (1 copy)

U.S. Naval Academy
Naval Postgraduate School
Physics Department
Annapolis, Maryland
Attn: Prof. L. E. Kinsler (1 copy)

Director,
Marine Physical Laboratory
University of California
San Diego 52, California (1 copy)

Director,
U.S. Navy Underwater Sound Laboratory
Fort Trumbull
New London, Connecticut (1 copy)

Director,
David Taylor Model Basin
Carderock, Maryland
Attn: Sound Section (1 copy)

Director,
Ordnance Research Laboratory
Penn. State College
State College, Pennsylvania (1 copy)

Chief of the Bureau of Ships
Navy Department
Washington 25, D. C.
Attn: Code 330 (1 copy)
845 (1 copy)
665 e (1 copy)

Naval Medical Research Institute
Naval Medical Center
Bethesda 14, Maryland
Attn: Lt. Cdr. D. Goldman NMC (1 copy)

Director,
Naval Ordnance Laboratory
White Oak, Silver Spring 19, Maryland
Attn: Sound Division (1 copy)

Department of the Air Forces

Commanding Officer
Air Force Cambridge Research Laboratories
230 Albany Street
Cambridge 39, Massachusetts
Attn: Geophysical Research Library (1 copy)

National Research Council

National Academy of Science
Committee on Undersea Warfare
2101 Constitution Avenue
Washington 25, D. C.
Attn: Dr. John S. Coleman (1 copy)

Director,
National Bureau of Standards
Division of Mechanics
Washington 25, D. C.
Attn: Dr. R. K. Cook (1 copy)

Los Alamos Scientific Laboratory
P.O. Box 1663
Los Alamos, New Mexico
Attn: Mr. Gilbert L. Campbell, Project Librarian (1 copy)

U.S. Atomic Energy Commission
Library Section, Technical Info. Branch, ORE
P.O. Box E
Oak Ridge, Tennessee
Attn: Dr. I. A. Warheit (3 copies)

B. Non-Government Distribution

Director,
Laboratory of Acoustic War Research
Penn. State College
State College, Pennsylvania (1 copy)

Mass. Institute of Technology
Acoustics Laboratory
Cambridge 39, Mass.
Attn: Prof. R. H. Bolt (1 copy)

Harvard University
Cruft Laboratory
Department of Engineering Science and Applied Physics
Cambridge, Mass.
Attn: Prof. F. V. Hunt (1 copy)

Catholic University of America
Washington 17, D. C.
Attn: Prof. F. K. Herzfeld (1 copy)

Brown University
Department of Applied Physics
Providence 12, Rhode Island
Attn: Prof. R. B. Lindsay (1 copy)

University of Southern California
Los Angeles, California
Attn: Prof. R. E. Vollrath (1 copy)

University of California Radiation Laboratory
Information Division
Room 128, Building 50
Berkeley, California
Attn: Dr. R. K. Wakerling (1 copy)

University of California
Department of Physics
Los Angeles, California
Attn: Dr. R. W. Leonard (1 copy)

Princeton University
Department of Electrical Engineering
Princeton, New Jersey
Attn: Dr. W. G. Johnson (1 copy)

Case Institute of Technology
Department of Physics
University Circle
Cleveland 6, Ohio
Attn: R. S. Shankland (1 copy)

Western Reserve University
Department of Chemistry
Cleveland, Ohio
Attn: Dr. F. Hovorka (1 copy)

Utah University
Salt Lake City 1, Utah
Attn: Dr. Elsey (1 copy)

8/2 STI-ATI-208 429 373920 UNCLASSIFIED

25/2,3 Scott Lab of Physics, Wesleyan U, Middletown, Conn

P9/1

A GENERALIZED THEORY OF THE CRYSTAL TRANSMITTER AND RECEIVER FOR PLANE WAVES, by Walter G. Cady. Tech Rpt No. 8. 20 Nov 50, 40p incl illus. (Contract N6onr-262, T.O. 1)

xElectroacoustic

Transducers

AD-A800 027

	SUBJECT HEADINGS
DIV: Electronics & Electronic Equipment (8)	Transducers, Electro-acoustic
SEC: Components & Accessories (2)	Crystals, Piezoelectric
DIV: Physics (25)	Crystals - Acoustic properties
SEC: Crystallography (2), Electricity & Magnetism (3)	Crystals - Electric properties

~~Copies obtainable from ASTIA-DSC~~ UNCLASSIFIED

CFSTI, auth: ONR memo, 3 Jan 68

8/2 STI-ATI-208 429 UNCLASSIFIED

25/2,3 Scott Lab of Physics, Wesleyan U, Middletown, Conn

A GENERALIZED THEORY OF THE CRYSTAL TRANSMITTER AND RECEIVER FOR PLANE WAVES, by Walter G. Cady. Tech Rpt No. 8. 20 Nov 50, 40p incl illus. (Contract N6onr-262, T.O. 1)

DIV: Electronics & Electronic Equipment (8)

SEC: Components & Accessories (2)

DIV: Physics (25)

SEC: Crystallography (2), Electricity & Magnetism (3)

SUBJECT HEADINGS

Transducers, Electro-acoustic

Crystals, Piezoelectric

Crystals - Acoustic properties

Crystals - Electric properties

(Copies obtainabl m ASTIA-DSC) UNCLASSIFIED